8.6 Solving Logarithmic Equations

A Exponential-Logarithmic Conversion The following two expressions are equivalent: $b^x = y \iff x = \log_b y$ $b > 0, b \neq 1; \quad y > 0, x \in R$	Ex 1. Solve for x. Verify restrictions. a) $\log x = 0$ b) $\ln x = 1$ c) $\log_2(x-1) = 0$ d) $\log(x^2+1) = 1$ e) $\ln(\log x) = 0$
B One-to-one property The logarithmic function is a one-to-one function. So: $\log_b x = \log_b y \iff x = y$ $b > 0, b \neq 1, x > 0, y > 0$	Ex 2. Solve for x. Verify restrictions. a) $\log(x-1) = \log(2x+1)$ b) $\ln(x+1) - \ln(x-1) = 3$ c) $\log_2(x-1) + \log_2(x+2) - \log_2(2x-1) = 1$ d) $\log x = 1 - \log(x-3)$
C Technology	Ex 3. Use technology (scientific calculator) to find the solution of the following equation to the nearest thousandth. $\ln x + \log x = 5$

Ex 4. Solve for x.

a) $\log_2(x^2) = (\log_2 x)^2$

b) $(\log x)^2 - \log x^2 + 1 = 0$

c) $\log_{x-1}(4x-4) = 2$

d) $\log_2(x-4) + \log_{\sqrt{2}}(x^3-2) + \log_{0.5}(x-4) = 20$

e) $4\log\sqrt{x} - 5\sqrt{\log x} = 3$

D Inequalities and Logarithms

If $b > 1$ then:

$$\log_b x > \log_b y \quad \Leftrightarrow \quad x > y$$

If $b < 1$ then:

$$\log_b x > \log_b y \quad \Leftrightarrow \quad x < y$$

Ex 5. Solve each inequality.

a) $\log x > 1$

b) $\ln(x-1) < 0$

c) $\log_{0.5}(2x+1) \geq 2$

d) $\log_{0.1} x^2 \leq -1$

Reading: Nelson Textbook, Pages 487-490
Homework: Nelson Textbook, Page 491: #4acf, 5ace, 6, 7ad, 8, 10, 12, 16, 18, 19, 20

